NPS71-87-002

NAVAL POSTGRADUATE SCHOOL

Monterey, California

ACTIVE SONAR DETECTION AND SIGNAL EXCESS FLUCTUATIONS

by

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NOVEMBER 1987

Approved for public release; distribution unlimited.

Prepared for:
Naval Air Development Center
Warminster, PA 18974

NAVAL POSTGRADUATE SCHOOL
Monterey, California

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UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution unlimited.
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S) NPS71-87-002	5. MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION Naval Postgraduate School 6b. OFFICE SYMBOL (If applicable) Code 71	7a. NAME OF MONITORING ORGANIZATION
6c. ADDRESS (City, State, and ZIP Code) Monterey, CA 93943-5000	7b. ADDRESS (City, State, and ZIP Code)
8a. NAME OF FUNDING/SPONSORING ORGANIZATION Naval Air Development Center 8b. OFFICE SYMBOL (If applicable) Code 3C1	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER N62269/87/WR00135
8c. ADDRESS (City, State, and ZIP Code) Warminster, PA 18974	10. SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO. 65155N PROJECT NO. 3C770100 TASK NO. ITDA WORK UNIT ACCESSION NO.

11. TITLE (Include Security Classification)
ACTIVE SONAR DETECTION AND SIGNAL EXCESS FLUCTUATIONS

12. PERSONAL AUTHOR(S)
Forrest, R. N.

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
Technical	FROM ______ TO ______	1987, November	52

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	active sonar detection models, signal excess fluctuation models

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

The report contains an analysis of an active sonar detection model. A k-out-of-n positive signal excess criterion determines echo detection. The analysis is based on encounter simulations in which signal excess fluctuations are described by either a Gauss-Markov process or a lambda-sigma jump process. For both processes, encounter probability estimates generated by the simulations suggest that the probability of detection during an encounter is a maximum when the relaxation time for the process is in the neighborhood of the time required to generate k consecutive pings given the time between pings remains constant.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL R. N. Forrest	22b. TELEPHONE (Include Area Code) (408)646-2653 22c. OFFICE SYMBOL Code 71

DD FORM 1473, 84 MAR — 83 APR edition may be used until exhausted. All other editions are obsolete

SECURITY CLASSIFICATION OF THIS PAGE

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I. Introduction

This report describes an analysis of a model of the active sonar detection process. The analysis was done in order to estimate the effects of signal excess fluctuations on the probability of detecting a sonar echo. The analysis is an extension of an analysis that is described in Reference 1.

The model has the following characteristics: Each return corresponding to a resolution cell has an associated signal excess. A return is conditionally classified as an echo if and only if the signal excess associated with the return is greater than or equal to zero (a success). A return is unconditionally classified as an echo if and only if it is the k^{th} success in a sequence of no greater than n returns. This detection criterion has been referred to as a k-out-of-n criterion.

For a resolution cell that does not contain a target, the signal excess is negative infinity. For a resolution cell that does contain a target, the signal excess is a random quantity that is determined by a stochastic process. Three stochastic processes were considered in the analysis. The first is the gauss-markov process, the second is the lambda-sigma jump process and the third is a modification of this process. In the analysis that is described in Reference 1, only the lambda-sigma jump process and its modification were considered. A discussion of the gauss-markov process which supports its inclusion in the analysis is contained in Reference 2 and an example of its use in an active sonar detection model is contained in Reference 3.

The active sonar detection models that are described in Reference 3, Reference 4 and Reference 5 are all based on a k-out-of-n criterion. In Reference 5, it is suggested that k equal to three and n equal to five give an adequate description of the active sonar detection process in which an operator determines the classification of the sonar returns. In Reference 6, arguments based on operational data are given both for determining sonar return classification in terms of signal excess and for a k-out-of-n criterion.

Since each return corresponding to a resolution cell has an associated signal excess, the model can accommodate false contacts based on signal inputs but not false contacts based on noise alone. However, if in the model the probability of conditionally classifying noise as an echo were .001, then with a 3-out-of-5 criterion the chance of a false contact based on noise alone would be less than one in ten million.

II. The Active Sonar Detection Model

In the model of the active sonar detection process, the classification of an echo is determined by its signal excess. A sonar return is conditionally classified as an echo if and only if the signal excess is greater than or equal to zero. The signal excess is the difference between the echo signal-to-noise ratio in decibels and the sonar receiver recognition differential, both of which are independent random variables in the model. With $X_{SE}(t)$ the signal excess for an echo generated at a time indicated by the index t, the signal excess is:

(1) $X_{SE}(t) = 10 \log(S/N) - X_{RD}(t)$

where $10 \log(S/N)$ is the signal-to-noise ratio in decibels for an echo generated at that time and $X_{RD}(t)$ is the recognition differential for the echo. The random signal-to-noise ratio in decibels is determined by the following sonar equation:

(2) $10 \log(S/N) = X_{SL}(t) - 2\,X_{TL}(t) + X_{TS}(t) - [X_{NL}(t) - X_{DI}(t)]$

where, $X_{SL}(t)$ is the sonar source level, $X_{TL}(t)$ is the one-way transmission loss between the target and the sonar, $X_{TS}(t)$ is the target strength, $X_{NL}(t)$ is the noise level and $X_{DI}(t)$ is the sonar directivity index. The random variable $X_{NL}(t)$ accounts for ambient noise, self noise and reverberation and represents a power sum of these quantities.

From Equations 1 and 2, the mean value of the signal excess is given by:

(3) $SE(t) = SL(t) - 2\,TL(t) + TS(t) - [NL(t) - DI(t)] - RD(t)$

where each term represents the mean value of its corresponding

random variable. In the model, the mean values are the values that would be used in the sonar equation that Equation 3 usually represents. From Equation 1 and Equation 3 the signal excess can be expressed as follows:

(4) $X_{SE}(t) = SE(t) + X(t)$

where $X(t)$ is a random variable whose mean is zero and whose variance is equal to the variance of $X_{SE}(t)$. Equation 4 implies that for each index t that is involved in describing an encounter there is a random variable $X(t)$ that determines the fluctuations in the signal excess at the time corresponding to the index t.

The three stochastic processes that were used in the analysis to determine the random variables represented by $X(t)$ are described in Section III. Unfortunately, it appears that the following quotation from Reference 7 has some relevance for each of the processes: "As discussed in Volume I, it is common practice in performance analyses to model signal or noise or signal excess fluctuations as stochastic processes. The choice of a specific process may be based on experience but usually tends toward mathematical convenience."

III. Two Simulation Programs

Two simulation programs for an IBM-PC were used in the analysis of the active sonar detection model. The two programs differ in the stochastic process that determines the random components of the signal excess. In the first program, the stochastic process is either a lambda-sigma jump process or a modification of a lambda-sigma jump process. In the second program, the stochastic process is a gauss-markov process. The programs which are written in BASICA are described in Appendix 1 and are listed in Appendix 2.

The programs simulate encounters between a submarine and an active sonar. In an encounter, the sonar source level, the transmission loss, the submarine target strength, the noise level and the sonar directivity index are independent of azimuth. In addition, the course, speed and depth of the submarine and the depth of the transmitting and receiving hydrophones of the sonar are constant throughout an encounter. This implies that the track of the submarine relative to the sonar is a straight line and that the encounter can be determined by specifying following quantities: the depth of the submarine, the depth of the transmitting hydrophone, the depth of the receiving hydrophone, the relative speed of the submarine, the horizontal range of the submarine at its closest point of approach (CPA), the time at the beginning of the encounter and the time at the end of the encounter. An encounter of this kind that begins and ends at ranges beyond which the probability of detecting the submarine is

essentially zero is called a complete straight line encounter. For a complete straight line encounter, only the horizontal range at CPA is required to determine the encounter geometry. This range is called the lateral range. The probability that detection occurs during a complete straight line encounter can be expressed as a function of the lateral range. The function is called the lateral range function or lateral range curve.

The lateral range functions plots that are in Section IV were generated with the two simulation programs. The mean transmission loss values that were used to do this are plotted in Figure 1 of Section IV. The values correspond to horizontal ranges from 2000 meters to 100,000 meters in 2000 meter increments. For a target range that is not an integer multiple of 2000 meters, the mean transmission loss value that is used in the programs is the value corresponding to the greatest integer multiple that is less than the target range. Relative to the programs, the transmission loss is determined by a step function with a step every 2000 meters.

Because of the step function nature of the transmission loss, the relative track of the target in an encounter is divided into sectors such that when the target is within a sector the transmission loss is constant. The simulation programs generate estimates of both the conditional and the unconditional probability that detection occurs on a sector.

Clearly, the encounters represented by the simulation programs are abstractions. In particular, except for the mean

transmission loss $TL(t)$, all terms on the right side of Equation 2 are constant during an encounter. Changes due to orientation, location and relative motion are ignored. For example, in addition to ignoring the change in the noise field with azimuth, the change in the noise field due to the doppler gain that results from the radial motion of the sonar and the submarine is also ignored. (More detailed models are described in Reference 3, Reference 4 and Reference 5.) However, the simulation programs were assumed to be adequate to satisfy the purpose of the analysis.

Because of the conditions described above, Equation 3 can be written as:

(5) $SE(t) = SL - 2\ TL[r(t)] + TS - (NL - DI) - RD$

where the transmission loss at a time corresponding to the index t is determined by the sonar transmitting hydrophone depth, the sonar receiving hydrophone depth, the submarine depth and $r(t)$, the submarine horizontal range at that time.

Both the lambda-sigma jump process and the gauss-markov process have been used to predict operational performance of sonar systems. This is not the case for the modified lambda-sigma jump process. The modification was introduced in order to deal with the observations that are reported in Reference 8. A modification based on a log-normal distribution rather than a shifted rayleigh distribution would have accomplished this also. The choice of a rayleigh distribution was based on mathematical convenience.

For a rayleigh random variable Y, the density function of its distribution is: $f_Y(y) = 2 \cdot \alpha \cdot y \cdot \exp(-\alpha \cdot y^2)$ where $y \geq 0$. With $\beta = \Gamma(3/2)$, the mean of the distribution is: $\mu_Y = \beta/\alpha^{\frac{1}{2}}$ and the variance is: $\sigma_Y^2 = (1 - \beta^2)/\alpha$.

The modified lambda-sigma jump process is defined as follows: If $X(t)$ is a random variable that is determined by a modified lambda-sigma jump process, then $X(t) = \mu_Y - Y_{N(t)}$ where $N(t)$ is determined by a Poisson process with a mean rate lambda and the random variables $Y_1, Y_2, \cdots$ are independent rayleigh random variables each with parameter α. This implies that $\mu_X = 0$ and $\sigma_X = \sigma_Y$. If $\alpha = (1 - \beta^2) \cdot (1/\sigma^2)$, then $\sigma_X = \sigma$. Consequently, if the random component of the signal excess is determined by a modified lambda-sigma jump process, then the standard deviation of the signal excess can be made equal to sigma by requiring that $\alpha = (1 - \beta^2) \cdot (1/\sigma^2)$.

The correlation coefficient between two random variables is a measure of their dependence. For the random variables $X(t_i)$ and $X(t_j)$ which represent the random component of the signal excess at times t_i and t_j and for $t_j > t_i$, the correlation coefficient has the form $\exp[-(t_j - t_i)/\tau]$ for the gauss-markov process, the lambda-sigma jump process and the modified lambda-sigma jump process. For each process, a value for the parameter τ and a value for the parameter σ are sufficient to define the process. The parameter τ is referred to as the relaxation time. It is the reciprocal of the parameter lambda in the lambda-sigma jump processes. For both the lambda-sigma jump

jump processes. For both it and the modified lambda-sigma jump process, τ is the expected time between jumps. If $\tau = 0$ for any one of the processes, the random components of the signal excess are independent. Otherwise, the random components are dependent. If $(1/\tau) = 0$ for any of the processes, the random components are completely dependent. This implies that the random components of the signal excess associated with an encounter all have the same value. Consequently, if the value of one of the random components of the signal excess is determined at some point in the encounter, the value of all of the other random components are also determined. Reference 9 contains a discussion of factors affecting relaxation times.

An acceptable choice for σ, the standard deviation of the signal excess, appears to be more easily agreed upon than an appropriate choice for τ, the relaxation time. However, the following quotation from Reference 10 is relevant both to these choices and to the choice of an appropriate stochastic process: "As of this date [1974] there is no unanimity of opinion as to which process [lambda-sigma jump or gauss-markov] to use in a detection model or what values of the parameters lambda and sigma to use. It is not even clear (certainly not to everyone) that this kind of detection model is uniformly valid. Some half-hearted attempts at `validating' this kind of model have been made but the results can be characterized as at best fragmentary and inconclusive. In my opinion, what is needed is a full-blown exercise whose sole purpose is to investigate the validity of

various hypothesized models for detection. Past attempts at this have merely been `ex post facto' exhumations of some sketchy exercise data."

IV. Simulation Results

Encounter simulation data are presented in this section. The data which are presented as lateral range function plots and as range sector probability of detection tables are from simulations that are based on the propagation loss values that are plotted in Figure 1, the encounter values that are listed in Table 1 and a 3-out-of-5 criterion.

For the simulated encounters, the lateral range function plots in Figure 2 through Figure 6 indicate the effect on the encounter probability of detection of the transmission loss function, especially the increase with range because of the convergence zone component. But, in particular, the figures indicate the effect on the encounter probability of detection of a gauss-markov process relative to that a lambda-sigma jump process for various relaxation times. Figure 7 is representative of the effect on encounter detection probability of a modified lambda-sigma jump process relative to that of the unmodified process. For additional comparison plots for the same simulated encounters, see Reference 1.

Except for the CPA range sector, each range sector is associated with two track segments that are symmetric about the CPA. The CPA range sector is associated with a single track segment that is bisected by the CPA. Table 2 and Table 3 list the probability of detection estimates on a track segment as a function of the track segment's distance from CPA and the range

sector associated with the track segment. Table 4 and Table 5 do this for the conditional probability of detection estimates. The values in Table 2 and Table 3 are estimates of the probability that detection will occur on a track segment. The values in Table 4 and in Table 5 are estimates of the probability that detection will occur on a track segment given detection has not already occurred on an earlier track segment.

The length of the simulated straight line encounters that were used in the analysis was restricted by the requirement that the range from the sonar at the end points of an encounter be less than or equal to 100 kilometers. Because of this restriction, the encounter track length is 120 kilometers for an encounter lateral range of 80 kilometers and the encounter track length is approximately 200 kilometers for an encounter lateral range of 10 kilometers. However, for the simulated acoustic conditions, the transmission loss for a range of 100 kilometers is 94.2 dB. If transmission loss values for ranges greater than 100 kilometers were constant and equal to 94.2 dB rather than generally increasing, for an echo from a target that was at a range greater than 100 kilometers, the probability of conditionally classifying the echo as an echo would be in the neighborhood of .000015. Thus, increasing the length of the simulated encounters should only make a negligible increase in the value of the encounter probability of detection estimates.

The probability estimates generated by the programs represent samples from binomial distributions to the degree of

randomness of the program simulations. However, the sample size of the estimates is not constant. In a simulated encounter, the encounter is terminated when a detection occurs. Consequently, for a given number of repetitions of an encounter simulation, the sample size for a range sector depends on the number of detections that have occurred in track segments prior to the track segment associated with the range sector. Because of this, each simulated encounter was repeated 1000 times in order to generate the lateral range function plot and sector probability of detection estimates. This number of samples is statistically sufficient for the purpose of the analysis reported here.

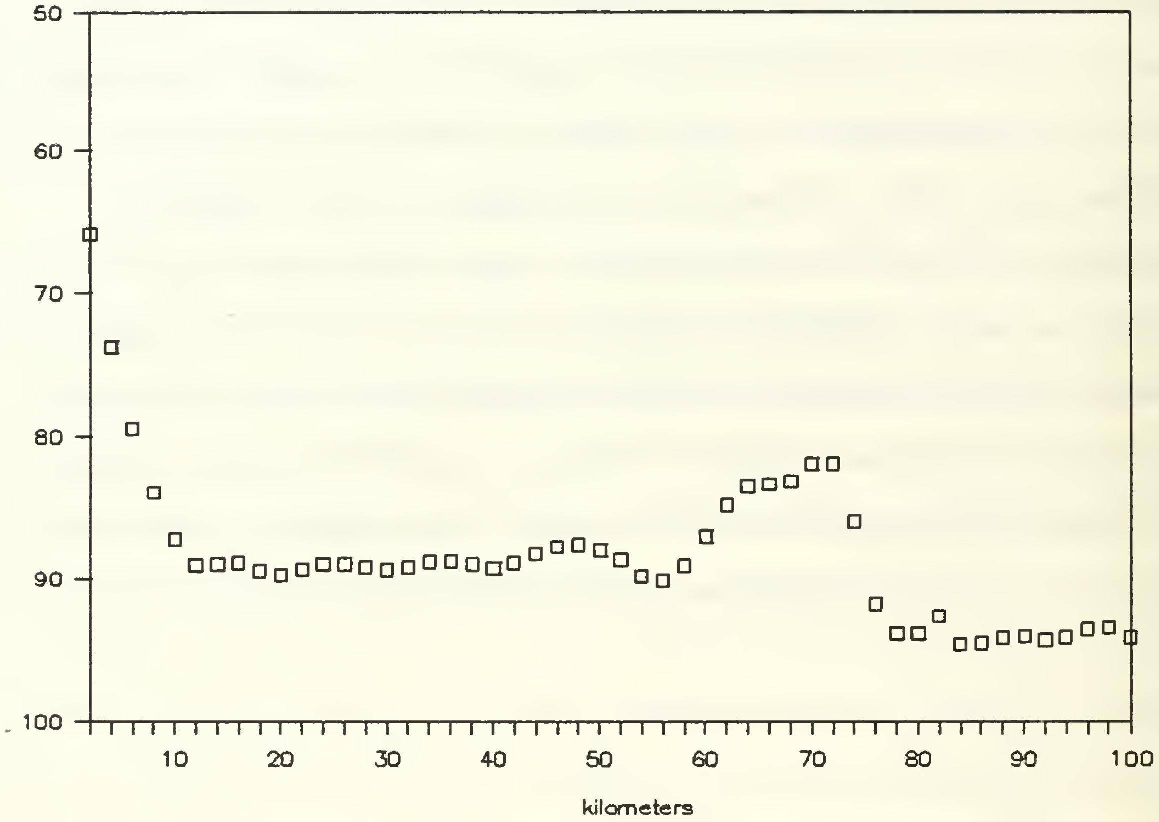

Figure 1. A plot of the encounter transmission loss values in dB against their corresponding ranges in kilometers. The values are listed in Appendix 4.

Recognition Differential	20 dB
Source Level	215 dB
Target Strength	5 dB
Noise Level	65 dB
Directivity Index	20 dB
Standard Deviation: σ	8 dB
Ping Cycle Time	61 seconds

Table 1. The parameter values for the simulated encounters.

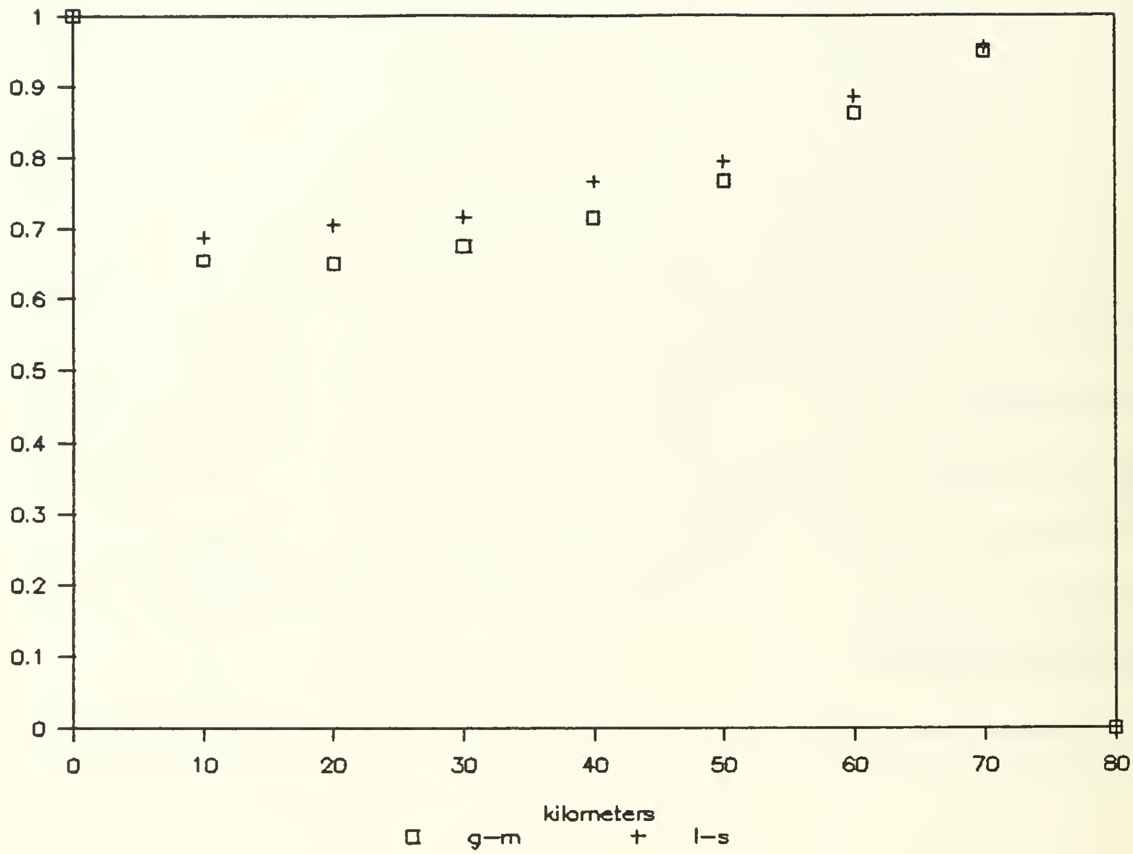

Figure 2. Lateral range function plots for a gauss-markov process and a lambda-sigma jump process for a relaxation time τ of 3 minutes.

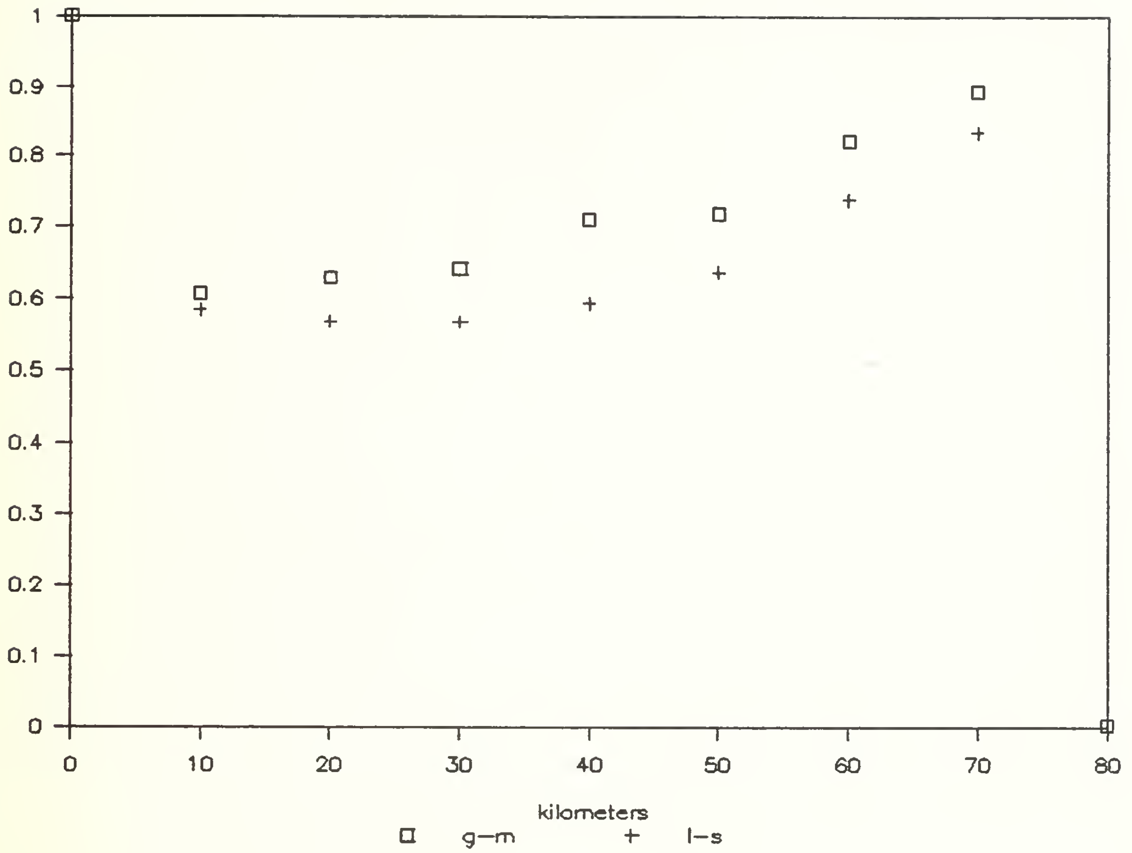

Figure 3. Lateral range function plots for a gauss-markov process and a lambda-sigma jump process for a relaxation time τ of 10 minutes.

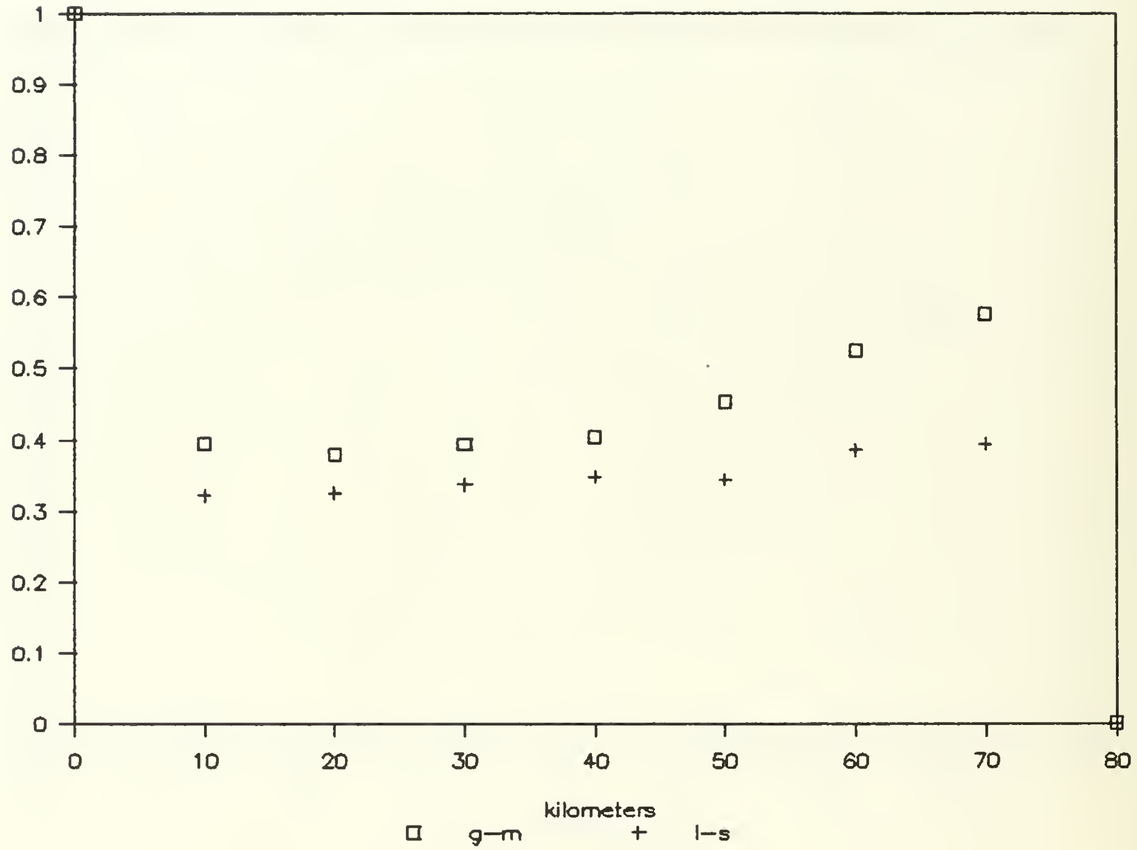

Figure 4. Lateral range function plots for a gauss-markov process and a lambda-sigma jump process for a relaxation time τ of 60 minutes.

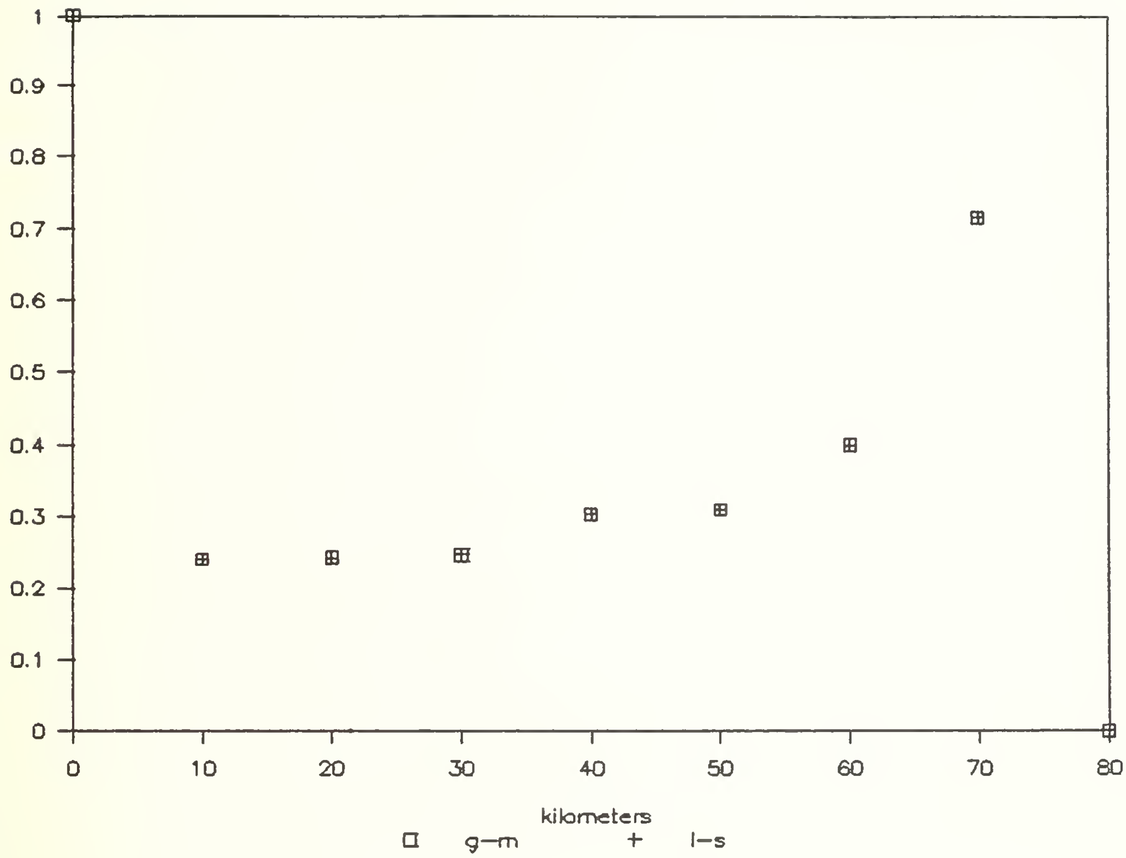

Figure 5. Lateral range function plots for a gauss-markov process and a lambda-sigma jump process for a relaxation time τ of zero.

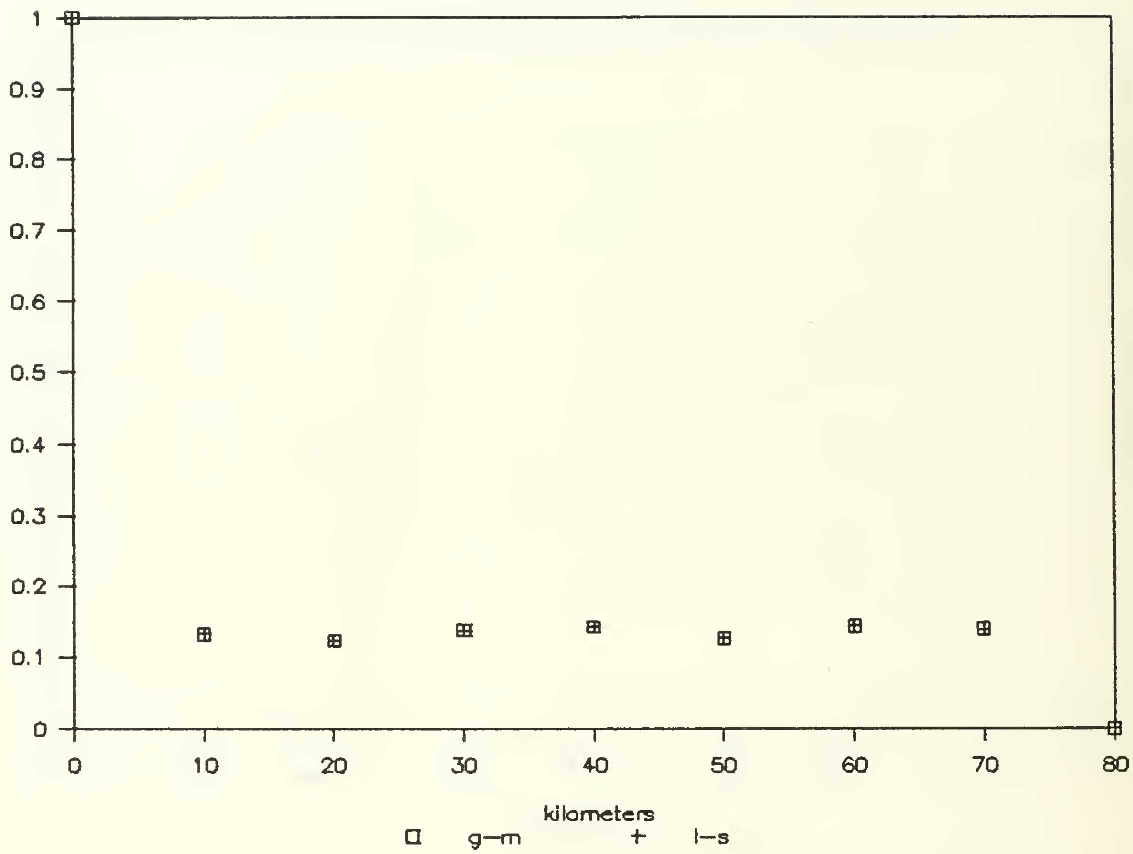

Figure 6. Lateral range function plots for a gauss-markov process and a lambda-sigma jump process for a relaxation time τ of infinity.

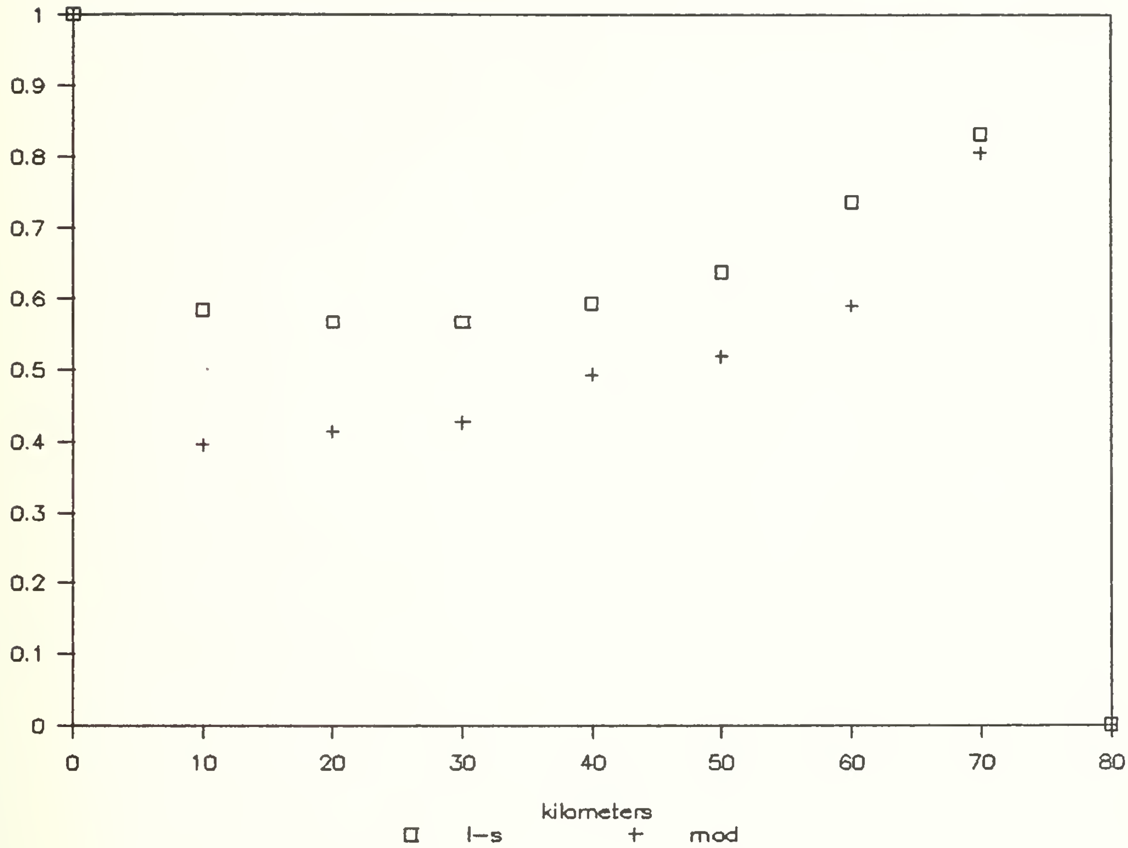

Figure 7. Lateral range function plots for a lambda-sigma jump process and a modified lambda-sigma jump process for a relaxation time τ of 10 minutes.

Table 2. Range sector probability of detection estimates for a gauss-markov process, a lateral range of 10,000 meters, a relaxation time τ of 3 minutes and a 3-out-of-5 criterion. Range sector and distance from CPA values are in meters and zero probability values are not listed.

probability	range sector	distance from CPA
.01	76000	-74330
.132	74000	-72312
.089	72000	-70292
.062	70000	-68272
.029	68000	-66250
.045	66000	-64226
.007	64000	-62201
.003	62000	-60175
.001	50000	-47969
.001	48000	-45924
.001	40000	-37697
.002	38000	-35623
.001	26000	-22913
.007	12000	-4583
.007	10000	0
.001	12000	4583
.001	32000	29343
.001	38000	35623
.001	46000	43875
.002	50000	47969
.001	52000	50010
.001	58000	56116
.002	60000	58146
.007	62000	60175
.040	64000	62201
.030	66000	64226
.046	68000	66250
.060	70000	68272
.054	72000	70292
.009	74000	72312

Table 3. Range sector probability of detection estimates for a lambda-sigma jump process, a lateral range of 10,000 meters, a relaxation time τ of 3 minutes and a 3-out-of-5 criterion. Range sector and distance from CPA values are in meters and zero probability values are not listed.

probability	range sector	distance from CPA
.01	76000	-74330
.147	74000	-72312
.090	72000	-70292
.061	70000	-68272
.046	68000	-66250
.033	66000	-64226
.017	64000	-62201
.005	62000	-60175
.001	60000	-58146
.003	52000	-50010
.002	50000	-47969
.001	48000	-45924
.003	46000	-43875
.002	42000	-39762
.001	40000	-37697
.001	38000	-35623
.001	28000	-25080
.001	26000	-22913
.001	24000	-20712
.003	20000	-16155
.002	18000	-13748
.012	12000	-4583
.008	10000	0
.001	14000	8307
.001	16000	11180
.003	20000	16155
.001	22000	18466
.001	26000	22913
.003	28000	25080
.002	30000	27221
.001	32000	29343
.001	36000	35541
.001	40000	37696
.001	44000	41821
.003	46000	43875
.004	48000	45924
.002	50000	47969
.001	56000	54083
.005	58000	56116
.005	60000	58146
.010	62000	60175
.040	64000	62201
.029	66000	64226
.039	68000	66250
.043	70000	68272
.046	72000	70292
.005	74000	72312

Table 4. Range sector conditional probability of detection estimates for a gauss-markov process, a lateral range of 10,000 meters, a relaxation time of 3 minutes and a 3-out-of-5 criterion. Range sector and distance from CPA values are in meters and zero probability values are not listed.

probability	range sector	distance from CPA
.01	76000	-74330
.133	74000	-72312
.104	72000	-70292
.081	70000	-68272
.041	68000	-66250
.066	66000	-64226
.011	64000	-62201
.005	62000	-60175
.002	50000	-47969
.002	48000	-45924
.002	40000	-37697
.003	38000	-35623
.002	26000	-22913
.011	12000	-4583
.011	10000	0
.002	12000	4583
.002	32000	29343
.002	38000	35623
.002	46000	43875
.003	50000	47969
.002	52000	50010
.002	58000	56116
.003	60000	58146
.012	62000	60175
.068	64000	62201
.055	66000	64226
.089	68000	66250
.128	70000	68272
.132	72000	70292
.025	74000	72312

Table 5. Range sector conditional probability of detection estimates for a lambda-sigma jump process, a lateral range of 10,000 meters, a relaxation time τ of 3 minutes and a 3-out-of-5 criterion. Range sector and distance from CPA values are in meters and zero probability values are not listed.

probability	range sector	distance from CPA
.01	76000	-74330
.148	74000	-72312
.107	72000	-70292
.081	70000	-68272
.066	68000	-66250
.051	66000	-64226
.027	64000	-62201
.008	62000	-60175
.002	60000	-58146
.005	52000	-50010
.003	50000	-47969
.002	48000	-45924
.005	46000	-43875
.003	42000	-39762
.002	40000	-37697
.002	38000	-35623
.002	28000	-25080
.002	26000	-22913
.002	24000	-20712
.003	20000	-16155
.005	18000	-13748
.021	12000	-4583
.014	10000	0
.002	14000	8307
.002	16000	11180
.005	20000	16155
.002	22000	18466
.002	26000	22913
.006	28000	25080
.004	30000	27221
.002	32000	29343
.002	36000	35541
.002	40000	37696
.002	44000	41821
.006	46000	43875
.008	48000	45924
.004	50000	47969
.002	56000	54083
.010	58000	56116
.010	60000	58146
.020	62000	60175
.079	64000	62201
.063	66000	64226
.090	68000	66250
.109	70000	68272
.131	72000	70292
.016	74000	72312

V. Conclusions

For the simulated encounters, the lateral range function plots in Section IV suggest that the probability of detection during an encounter is a maximum for a relaxation time in the neighborhood of 3 minutes for the gauss-markov process as well as for the lambda-sigma jump process. A heuristic argument for this apparent relation between probability of detection, relaxation time and detection criterion for the lambda-sigma jump process is given in Appendix 3.

For the simulated encounters and a relaxation time of 3 minutes, the lateral range function plots in Section IV also suggest that the probability of detection during an encounter for the gauss-markov process and for lambda-sigma jump process are essentially equivalent. This apparent equivalence does not hold generally. For example, for relaxation times of 10 minutes and 60 minutes, the probability of detection during an encounter for the gauss-markov process is greater than that for the lambda-sigma jump process. However, for both processes, the lateral range function plots for a relaxation time of zero correspond to encounters in which the random components of the signal excess are independent gaussian random variables. And, for both processes, the lateral range function plots for a relaxation time of infinity correspond to encounters in which the random components of the signal excess are dependent gaussian random variables with correlation coefficients of one. Consequently, in the limit for a relaxation time of zero or infinity, the two

processes are equivalent. Figure 5 shows their lateral range plots for a relaxation time of zero and Figure 6 shows their lateral range plots for a relaxation time of infinity.

In Table 6 on Page 33, for the nominal encounter ping cycle time of 60 seconds and a lambda-sigma jump process, conditional probability of detection estimates are listed for a 1-out-of-1 detection criterion and a 3-out-of-5 detection criterion. (In a 1-out-of-1 detection criterion, a return is unconditionally classified as an echo if and only if the signal excess associated with the return is greater than or equal to zero.)

Also, in Table 6, conditional probabilities of detection are listed for an encounter with a single ping per sector, 1-out-of-1 detection criterion. For an encounter with a single ping per sector, 1-out-of-1 detection criterion, $\Phi(SE/\sigma)$ is the conditional probability of detection for a range sector where Φ indicates the standard normal (gaussian) cumulative distribution function. Conditional probabilities of detection determined for a range sector by $\Phi(SE/\sigma)$ have been used to construct probability of detection maps. Table 6 suggests that for a lambda-sigma jump process, the conditional probabilities of detection for a 3-out-of-5 detection criterion may not be significantly different from the values determined by $\Phi(SE/\sigma)$. However, in Appendix 3, a heuristic argument indicates that for a lambda-sigma jump process, a 3-out-of-5 encounter conditional probability of detection estimate is of the order of $\Phi(SE/\sigma)\cdot(t/\tau)\cdot\exp(-s/\tau)$ where s is the time required for 3

consecutive successes to occur and for this case t can be considered to be the time in a range sector. For the encounter simulation that generated the data in Table 6, both s and τ are equal to 3 minutes and, for the range sectors with reasonable sample sizes, t/τ is approximately equal to 2. This implies that the 3-out-of-5 values should be of the order of $\Phi(SE/\sigma)$ which is consistent with Table 6. For an encounter whose lateral range is 10 kilometers, the length of the range sector track segments go from zero for a range of 100 kilometers to 13266 meters for the CPA range of 10 kilometers. For the nominal ping cycle time of 60 seconds, the number of pings in the CPA range sector track segment is approximately 42. A table similar to Table 6 is generated for the gauss-markov process for the same encounter conditions.

Table 6 also suggests that, for a three minute relaxation time, the 1-out-of-1 criterion is superior to the 3-out-of-5 criterion because of the increased probability of detection and the minimum decision delay. However, for an operational sonar system, the false alarm probability would be significantly lower for the 3-out-of-5 criterion given typical single return false alarm probabilities.

single ping	1-out-of-1	3-out-of-5	range sector
.000			76000
.017	.047	.015	74000
.130	.323	.127	72000
.130	.192	.107	70000
.077	.180	.079	68000
.070	.119	.055	66000
.067	.149	.064	64000
.032	.028	.022	62000
.008	.026	.010	60000
.002	.003	.002	58000
.001	.007	.002	56000
.001	.007		54000
.002	.007	.002	52000
.004	.017	.005	50000
.005	.003	.005	48000
.005	.017	.003	46000
.003		.003	44000
.002	.007		42000
.002			40000
.002		.002	38000
.002	.007		36000
.002	.011	.002	34000
.002	.003	.002	32000
.001			30000
.002	.007		28000
.002	.011	.002	26000
.002	.011		24000
.001			22000
.001	.004		20000
.001		.005	18000
.002	.007		16000
.002	.007		14000
.002	.008	.005	12000
.007	.111	.022	10000

Table 6. Range sector conditional probabilities of detection for a single ping per sector, 1-out-of-1 criterion and conditional probability of detection estimates for a lambda-sigma jump process and a 1-out-of-1 criterion, a 3-out-of-5 criterion and a 10,000 meter lateral range. Range sector values are in meters and zero probability values are not listed.

Appendix 1. Program Descriptions

The programs that were used to compute encounter detection probability estimates are listed in Appendix 2. The programs are written in BASICA and were run on an IBM-PC using QuickBasic which is licensed by Microsoft Corporation.

When a program is run, an initial prompt provides a choice of either generating a file containing encounter probability estimates or printing the data contained in a previously generated file. When the program is used to generate encounter probability estimates, additional prompts request parameter values that are required to define an encounter.

Appendix 2. Program Listings

```
10 REM active gauss-markov lateral range function/range sector
probabilities
20 P$="RSGMA.BAS"
30 DIM TL(50),PP(20),N0(20),LR(20),CTA(20),M1(150),P1(150,20),
P2(150,20),R1(150,20),Y1(150,20)
40 PI=4*ATN(1)
50 INPUT "generate file or print file (g/p)";N$
60 IF N$="G" OR N$="g" THEN GOTO 230
70 IF N$="P" OR N$="p" THEN GOTO 80 ELSE GOTO 50
80 PRINT
90 ON ERROR GOTO 100: GOTO 110
100 RESUME 90
110 INPUT "file name";A$
120 OPEN "I",#1,A$
130 INPUT#1,RD,SL,TS,NL,DI,TAU,SIG,W,LRM,ST,CT,N3,N0,N8,M8,B$,
P$,L
140 FOR J=1 TO L
150 INPUT#1, PP(J),LR(J),M1(J),N0(J),CTA(J)
160 FOR I=0 TO M1(J)
170 INPUT#1, P1(I,J),R1(I,J),Y1(I,J)
180 NEXT I
190 NEXT J
200 CLOSE
210 ON ERROR GOTO 0
220 GOTO 1440
230 PRINT
240 INPUT "maximum transmission loss range (50000/100000)";MTLR
250 IF MTLR=50000! THEN RI=1000 ELSE IF MTLR=100000! THEN RI=2000
ELSE GOTO 230
260 INPUT "transmission loss data entry (K=keyboard/D=disk)";A$
270 IF A$="D" OR A$="d" THEN GOTO 400
280 IF A$="K" OR A$="k" THEN GOTO 290 ELSE GOTO 260
290 FOR I=1 TO 50
300 A$=STR$(I*2000)
310 PRINT "transmission loss at "A$" meters (dB)": INPUT TL(I)
320 NEXT I
330 TL(0)=TL(1)
340 INPUT "transmission loss file name";B$
350 OPEN "O",#1,B$
360 FOR I=0 TO 50
370 WRITE#1, TL(I)
380 NEXT I
390 CLOSE: GOTO 490
400 ON ERROR GOTO 410: GOTO 420
410 RESUME 400
420 INPUT "transmission loss file name";B$
430 OPEN "I",#1,B$
440 FOR I=0 TO 50
450 INPUT#1, TL(I)
460 NEXT I
```

```
470 CLOSE
480 ON ERROR GOTO 0
490 INPUT "recognition differential (dB)";RD
500 INPUT "source level (dB)";SL: INPUT "target strength (dB)";TS
510 INPUT "noise level (dB)";NL: INPUT "directivity index (dB)";
DI
520 FOM=SL+TS-NL+DI-RD: REM figure of merit
530 INPUT "relaxation time tau (minutes, -1 for infinity)";TAU
540 INPUT "sigma (dB)";SIG
550 INPUT "relative speed (knots)";W
560 IF W<=0 THEN PRINT "must be greater than 0": GOTO 550
570 W=W*1852/60: REM relative speed in meters per minute
580 INPUT "maximum lateral range (meters)";LRM
590 IF LRM>.8*MTLR THEN PRINT "maximum is "
+STR$(.8*MTLR)+" meters": GOTO 580
600 INPUT "lateral range step (meters)";ST
610 IF ST>LRM THEN PRINT "maximum step is "+STR$(LRM): GOTO 600
620 L=INT(LRM/ST)
630 IF L>20 THEN PRINT "minimum step is "+STR$(LRM/20): GOTO 600
640 INPUT "ping cycle time (seconds)";CT: CTM=300
650 IF CT>CTM THEN PRINT "must be less than "+STR$(CTM)+
" seconds": GOTO 640
660 CT=CT/60: REM ping cycle time in minutes
670 N5=INT(SQR(MTLR*MTLR-LRM*LRM)/W/CT): REM number of pings to
CPA
680 INPUT "pings in detection criterion window";N8
690 IF N8>=2*N5 THEN PRINT "must be less than "+STR$(2*N5):
GOTO 680
700 IF N8<1 THEN PRINT "minimum is 1 : GOTO 480
710 INPUT "echoes required for detection";M8
720 IF M8<1 THEN PRINT "minimum is 1": GOTO 710
730 IF M8>N8 THEN PRINT "maximum is "+STR$(N8): GOTO 710
740 INPUT "repetitions";N3
750 IF N3<=0 THEN PRINT "must be greater than 0": GOTO 740
760 INPUT "data file name";A$
770 LR=0
780 FOR J=1 TO L
790 LR=LR+ST: LR(J)=LR: REM lateral range
800 Y0=SQR(MTLR*MTLR-LR*LR): REM distance from CPA at encounter
start
810 TE=Y0/W: REM TE=time to CPA in minutes
820 N0=INT(TE/CT): IF N0=0 THEN N0=1: REM adjusted number of
pings to CPA
830 CTA=TE/N0: REM adjusted ping cycle time in minutes
840 DIM SE(2*N0),P(2*N0),Y(2*N0),R(2*N0),N(2*N0)
850 TK=-TE
860 FOR K=0 TO N0
870 R1=SQR(LR*LR+W*W*TK*TK): IN=INT(R1/RI) REM transmission loss
range index
880 SE=FOM-2*TL(IN)
890 SE(K)=SE: TK=TK+CTA: R(K)=IN*RI REM transmission loss range
for FOM
```

```
900 NEXT K
910 FOR K=N0+1 TO 2*N0
920 SE(K)=SE(2*N0-K): R(K)=R(2*N0-K)
930 NEXT K
940 P=0
950 FOR I=1 TO N3
960 DIM M(N8-1)
970 GOSUB 2120
980 X=SIG*Q
990 IF TAU =-1 GOTO 1020
1000 IF TAU=0 THEN RHO=0: FAC=1: GOTO 1020
1010 RHO=EXP(-CTA/TAU): FAC=SQR(1-RHO*RHO): REM gauss-markov
parameters
1020 FOR K=0 TO 2*N0
1030 IF K=0 OR TAU=-1 GOTO 1060
1040 GOSUB 2120
1050 X=SIG*FAC*Q+RHO*X
1060 XSE=SE(K)+X
1070 IF K<=N8-1 THEN J1=K: GOTO 1120
1080 FOR J1=0 TO N8-2
1090 M(J1)=M(J1+1)
1100 NEXT J1
1110 J1=N8-1
1120 IF XSE>=0 THEN M(J1)=1 ELSE M(J1)=0
1130 S=0
1140 FOR J1=0 TO N8-1
1150 S=S+M(J1)
1160 NEXT J1
1170 IF S>=M8 GOTO 1200
1180 NEXT K
1190 GOTO 1210
1200 P=P+1: P(K)=P(K)+1
1210 ERASE M
1220 NEXT I
1230 PP(J)=P/N3
1240 P1(0,J)=P(0): R1(0,J)=R(0): Y1(0,J)=-Y0
1250 FOR I=1 TO 2*N0
1260 IF R(I)=R(I-1) THEN N(I)=N(I-1): P1(N(I),J)=P(I)+P1(N(I),J):
GOTO 1310
1270 N(I)=N(I-1)+1: P1(N(I),J)=P(I): R1(N(I),J)=R(I)
1280 IF LR>R1(N(I),J) THEN Y1(N(I),J)=0: GOTO 1310
1290 Y1(N(I),J)=SQR(R1(N(I),J)*R1(N(I),J)-LR*LR)
1300 IF Y1(N(I),J)<ABS(Y1(N(I-1),J)) THEN Y1(N(I),J)=-Y1(N(I),J)
1310 NEXT I
1320 M1(J)=N(2*N0): N0(J)=N0: CTA(J)=CTA*60
1330 ERASE P,Y,R,N,SE
1340 NEXT J
1350 OPEN "O",#1,A$
1360 WRITE#1,RD,SL,TS,NL,DI,TAU,SIG,W,LRM,ST,CT,N3,N0,N8,M8,
B$,P$,L
1370 FOR J=1 TO L
1380 WRITE#1, PP(J),LR(J),M1(J),N0(J),CTA(J)
```

```
1390 FOR I=0 TO M1(J)
1400 WRITE#1, P1(I,J),R1(I,J),Y1(I,J)
1410 NEXT I
1420 NEXT J
1430 CLOSE
1440 PRINT
1450 INPUT "print encounter parameters (Y=yes/N=no)";D$
1460 IF D$="Y" OR D$="y" GOTO 1480
1470 IF D$="N" OR D$="n" GOTO 1660 ELSE GOTO 1450
1480 LPRINT
1490 LPRINT "program file name                             ",P$
1500 LPRINT "transmission loss file name                   ",B$
1510 LPRINT "recognition differential (dB)                 ",RD
1520 LPRINT "source level (dB)                             ",SL
1530 LPRINT "target strength (dB)                          ",TS
1540 LPRINT "noise level (dB)                              ",NL
1550 LPRINT "directivity index (dB)                        ",DI
1560 LPRINT "relaxation time tau (minutes)                 ",TAU
1570 LPRINT "sigma (dB)                                    ",SIG
1580 LPRINT "relative speed (knots)                        ",W*60/1852
1590 LPRINT "maximum lateral range (meters)                ",LRM
1600 LPRINT "lateral range step (meters)                   ",ST
1610 LPRINT "ping cycle time (seconds)                     ",CT*60
1620 LPRINT "pings in detection criterion window           ",N8
1630 LPRINT "echoes required for detection                 ",M8
1640 LPRINT "repetitions                                   ",N3
1650 LPRINT "data file name                                ",A$
1660 PRINT
1670 INPUT "print encounter probabilities (Y=yes/N=no)";D$
1680 IF D$="Y" OR D$="y" GOTO 1700
1690 IF D$="N" OR D$="n" GOTO 1780 ELSE GOTO 1670
1700 LPRINT: LPRINT: LPRINT: LPRINT A$+" encounter probabilities"
1710 FOR J=1 TO L
1720 LPRINT: LPRINT
1730 LPRINT "lateral range                                 ",LR(J)
1740 LPRINT "probability of detection                      ",PP(J)
1750 LPRINT "adjusted ping cycle time (seconds)            ",CTA(J)
1760 LPRINT
1770 NEXT J
1780 PRINT
1790 INPUT "print range sector probabilities (Y=yes/N=no)";D$
1800 IF D$="Y" OR D$="y" GOTO 1820
1810 IF D$="N" OR D$="n" GOTO 1930 ELSE GOTO 1790
1820 LPRINT: LPRINT: LPRINT: LPRINT A$+" range sector
probabilities"
1830 FOR J=1 TO L
1840 LPRINT: LPRINT
1850 LPRINT "lateral range                                 ",LR(J)
1860 LPRINT "probability of detection                      ",PP(J)
1870 LPRINT "adjusted ping cycle time (seconds)            ",CTA(J)
1880 LPRINT
1890 FOR I=0 TO M1(J)
```

```
1900 LPRINT "P("I") = "P1(I,J)/N3 TAB(30) "R("I") = "R1(I,J)
TAB(60) "Y("I") = "Y1(I,J)
1910 NEXT I
1920 NEXT J
1930 PRINT
1940 INPUT "print range sector conditional probabilities
(Y=yes/N=no)";D$
1950 IF D$="Y" OR D$="y" GOTO 1970
1960 IF D$="N" OR D$="n" GOTO 2110 ELSE GOTO 1940
1970 LPRINT: LPRINT: LPRINT: LPRINT A$+" range sector conditional
probabilities"
1980 FOR J=1 TO L
1990 LPRINT: LPRINT
2000 LPRINT "lateral range                              ",LR(J)
2010 LPRINT "probability of detection                   ",PP(J)
2020 LPRINT "adjusted ping cycle time (seconds)         ",CTA(J)
2030 LPRINT
2040 D=N3
2050 FOR I=0 TO M1(J)
2060 IF D=0 THEN P2(I,J)=0 ELSE P2(I,J)=P1(I,J)/D
2070 LPRINT "P("I") = "P2(I,J) TAB(30) "R("I") = "R1(I,J) TAB(60)
"Y("I") = "Y1(I,J)
2080 IF D>0 THEN D=D-P1(I,J)
2090 NEXT I
2100 NEXT J
2110 END
2120 RV=RND: IF RV=0 THEN RV=2.8E-38
2130 IF RV=1 THEN RV=1-2.8E-38
2140 Q=SQR(-2*LOG(RV))*SIN(2*PI*RND): REM standard normal random
number
2150 RETURN
```

```
10 REM active lambda-sigma lateral range function/range sector
probabilities
20 P$="RSLSA.BAS"
30 DIM TL(50),PP(20),N0(20),LR(20),CTA(20),M1(150),P1(150,20),
P2(150,20),R1(150,20),Y1(150,20)
40 PI=4*ATN(1): RC1=.88623: RC2=.21459
50 INPUT "generate file or print file (g/p)";N$
60 IF N$="G" OR N$="g" THEN GOTO 230
70 IF N$="P" OR N$="p" THEN GOTO 80 ELSE GOTO 50
80 PRINT
90 ON ERROR GOTO 100: GOTO 110
100 RESUME 90
110 INPUT "file name";A$
120 OPEN "I",#1,A$
130 INPUT#1, RD,SL,TS,NL,DI,TAU,SIG,W,LRM,ST,CT,N3,N0,N8,M8,
B$,C$,P$,L
140 FOR J=1 TO L
150 INPUT#1, PP(J),LR(J),M1(J),N0(J),CTA(J)
160 FOR I=0 TO M1(J)
170 INPUT#1, P1(I,J),R1(I,J),Y1(I,J)
180 NEXT I
190 NEXT J
200 CLOSE
210 ON ERROR GOTO 0
220 GOTO 1520
230 PRINT
240 INPUT "maximum transmission loss range (50000/100000)";MTLR
250 IF MTLR=50000! THEN RI=1000 ELSE IF MTLR=100000! THEN RI=2000
ELSE GOTO 230
260 INPUT "transmission loss data entry (K=keyboard/D=disk)";A$
270 IF A$="D" OR A$="d" GOTO 400
280 IF A$="K" OR A$="k" GOTO 290 ELSE GOTO 260
290 FOR I=1 TO 50
300 A$=STR$(I*2000)
310 PRINT "transmission loss at "A$" meters (dB)": INPUT TL(I)
320 NEXT I
330 TL(0)=TL(1)
340 INPUT "transmission loss file name";B$
350 OPEN "O",#1, B$
360 FOR I=0 TO 50
370 WRITE#1, TL(I)
380 NEXT I
390 CLOSE: GOTO 520
400 ON ERROR GOTO 410: GOTO 420
410 RESUME 400
420 INPUT "transmission loss file name";B$
430 OPEN "I",#1, B$
440 FOR I=0 TO 50
450 INPUT#1, TL(I)
460 NEXT I
470 CLOSE
480 ON ERROR GOTO 0
```

```
490 INPUT "signal excess distribution (G=gaussian/R=rayleigh)";C$
500 IF C$="G" OR C$="g" GOTO 520
510 IF C$="R" OR C$="r" GOTO 520 ELSE GOTO 490
520 INPUT "recognition differential (dB)";RD
530 INPUT "source level (dB)";SL: INPUT "target strength (dB)";TS
540 INPUT "noise level (dB)";NL: INPUT "directivity index (dB)";
DI
550 FOM=SL+TS-NL+DI-RD: REM figure of merit
560 INPUT "relaxation time tau (minutes, -1 for infinity)";TAU
570 INPUT "sigma (dB)";SIG
580 INPUT "relative speed (knots)";W
590 IF W<=0 THEN PRINT "must be greater than 0": GOTO 580
600 W=W*1852/60: REM relative speed in meters per minute
610 INPUT "maximum lateral range (meters)";LRM
620 IF LRM>.8*MTLR THEN PRINT "maximum is "+STR$(.8*MTLR)+
" meters": GOTO 610
630 INPUT "lateral range step (meters)";ST
640 IF ST>LRM THEN PRINT "maximum step is "+STR$(LRM): GOTO 630
650 L=INT(LRM/ST)
660 IF L>20 THEN PRINT "minimum step is "+STR$(LRM/20)+" meters":
GOTO 630
670 INPUT "ping cycle time (seconds)";CT: CTM=300
680 IF CT>CTM THEN PRINT "must be less than "+STR$(CTM)+
" seconds": GOTO 670
690 CT=CT/60: REM ping cycle time in minutes
700 N5=INT(SQR(MTLR*MTLR-LRM*LRM)/W/CT): REM number of pings to
CPA
710 INPUT "pings in detection criterion window";N8
720 IF N8>=2*N5 THEN PRINT "must be less than "+STR$(2*N5):
GOTO 710
730 IF N8<1 THEN PRINT "minimum is 1": GOTO 670
740 INPUT "echoes required for detection";M8
750 IF M8<1 THEN PRINT "minimum is 1": GOTO 740
760 IF M8>N8 THEN PRINT "maximum is "+STR$(N8): GOTO 740
770 INPUT "repetitions";N3
780 IF N3<=0 THEN PRINT "must be greater than 0": GOTO 770
790 INPUT "data file name";A$
800 LR=0
810 FOR J=1 TO L
820 LR=LR+ST: LR(J)=LR: REM lateral range
830 Y0=SQR(MTLR*MTLR-LR*LR): REM distance from CPA at encounter
start
840 TE=Y0/W: REM TE=time to CPA in minutes
850 N0=INT(TE/CT): IF N0=0 THEN N0=1: REM adjusted number of
pings to CPA
860 CTA=TE/N0: REM adjusted ping cycle time in minutes
870 DIM SE(2*N0),P(2*N0),Y(2*N0),R(2*N0),N(2*N0)
880 TK=-TE
890 FOR K=0 TO N0
900 R1=SQR(LR*LR+W*W*TK*TK): IN=INT(R1/RI) REM TL range index
910 SE=FOM-2*TL(IN)
```

```
920 SE(K)=SE: TK=TK+CTA: R(K)=IN*RI REM transmission loss range
for FOM
930 NEXT K
940 FOR K=N0+1 TO 2*N0
950 SE(K)=SE(2*N0-K): R(K)=R(2*N0-K)
960 NEXT K
970 P=0
980 FOR I=1 TO N3
990 DIM M(N8-1)
1000 IF TAU=0 GOTO 1050
1010 GOSUB 2210
1020 IF TAU=-1 GOTO 1050
1030 GOSUB 2270
1040 N=-TAU*LOG(1-RV)/CTA: REM number of pings to the first jump
1050 FOR K=0 TO 2*N0
1060 IF TAU=0 GOTO 1110
1070 IF K<N OR TAU=-1 GOTO 1120
1080 GOSUB 2270
1090 DN=-TAU*LOG(1-RV)/CTA
1100 N=N+DN: REM number of pings to the next jump
1110 GOSUB 2210
1120 XSE=SE(K)+Q
1130 IF K<=N8-1 THEN J1=K: GOTO 1180
1140 FOR J1=0 TO N8-2
1150 M(J1)=M(J1+1)
1160 NEXT J1
1170 J1=N8-1
1180 IF XSE>=0 THEN M(J1)=1 ELSE M(J1)=0
1190 S=0
1200 FOR J1=0 TO N8-1
1210 S=S+M(J1)
1220 NEXT J1
1230 IF S>=M8 THEN GOTO 1260
1240 NEXT K
1250 GOTO 1270
1260 P=P+1: P(K)=P(K)+1
1270 ERASE M
1280 NEXT I
1290 PP(J)=P/N3
1300 N(0)=0: P1(0,J)=P(0): R1(0,J)=R(0): Y1(0,J)=-Y0
1310 FOR I=1 TO 2*N0
1320 IF R(I)=R(I-1) THEN N(I)=N(I-1): P1(N(I),J)=P(I)+P1(N(I),J):
GOTO 1370
1330 N(I)=N(I-1)+1: P1(N(I),J)=P(I): R1(N(I),J)=R(I)
1340 IF LR>R1(N(I),J) THEN Y1(N(I),J)=0: GOTO 1370
1350 Y1(N(I),J)=SQR(R1(N(I),J)*R1(N(I),J)-LR*LR)
1360 IF Y1(N(I),J)<ABS(Y1(N(I-1),J)) THEN Y1(N(I),J)=-Y1(N(I),J)
1370 NEXT I
1380 M1(J)=N(2*N0): N0(J)=N0: CTA(J)=CTA*60
1390 ERASE P,Y,R,N,SE
1400 NEXT J
1410 IF C$="G" OR C$="g" THEN C$="gaussian": GOTO 1430
```

```
1420 C$="rayleigh"
1430 OPEN "O",#1,A$
1440 WRITE#1, RD,SL,TS,NL,DI,TAU,SIG,W,LRM,ST,CT,N3,N0,N8,
M8,B$,C$,P$,L
1450 FOR J=1 TO L
1460 WRITE#1, PP(J),LR(J),M1(J),N0(J),CTA(J)
1470 FOR I=0 TO M1(J)
1480 WRITE#1, P1(I,J),R1(I,J),Y1(I,J)
1490 NEXT I
1500 NEXT J
1510 CLOSE
1520 PRINT
1530 INPUT "print encounter parameters (Y=yes/N=no)";D$
1540 IF D$="Y" OR D$="y" GOTO 1560
1550 IF D$="N" OR D$="n" GOTO 1750 ELSE GOTO 1530
1560 LPRINT
1570 LPRINT "program file name                        ",P$
1580 LPRINT "transmission loss file name              ",B$
1590 LPRINT "signal excess distribution               ",C$
1600 LPRINT "recognition differential (dB)            ",RD
1610 LPRINT "source level (dB)                        ",SL
1620 LPRINT "target strength (dB)                     ",TS
1630 LPRINT "noise level (dB)                         ",NL
1640 LPRINT "directivity index (dB)                   ",DI
1650 LPRINT "relaxation time tau (minutes)            ",TAU
1660 LPRINT "sigma (dB)                               ",SIG
1670 LPRINT "relative speed (knots)                   ",W*60/1852
1680 LPRINT "maximum lateral range (meters)           ",LRM
1690 LPRINT "lateral range step (meters)              ",ST
1700 LPRINT "ping cycle time (seconds)                ",CT*60
1710 LPRINT "pings in detection criterion window      ",N8
1720 LPRINT "echoes required for detection            ",M8
1730 LPRINT "repetitions                              ",N3
1740 LPRINT "data file name                           ",A$
1750 PRINT
1760 INPUT "print encounter probabilities (Y=yes/N=no)";D$
1770 IF D$="Y" OR D$="y" GOTO 1790
1780 IF D$="N" OR D$="n" GOTO 1870 ELSE GOTO 1770
1790 LPRINT: LPRINT: LPRINT: LPRINT A$+" encounter probabilities"
1800 FOR J=1 TO L
1810 LPRINT: LPRINT
1820 LPRINT "lateral range                            ",LR(J)
1830 LPRINT "probability of detection                 ",PP(J)
1840 LPRINT "adjusted ping cycle time (seconds)       ",CTA(J)
1850 LPRINT
1860 NEXT J
1870 PRINT
1880 INPUT "print range sector probabilities (Y=yes/N=no)";D$
1890 IF D$="Y" OR D$="y" GOTO 1910
1900 IF D$="N" OR D$="n" GOTO 2020 ELSE GOTO 1880
1910 LPRINT: LPRINT: LPRINT: LPRINT A$+" range sector
probabilities"
```

```
1920 FOR J=1 TO L
1930 LPRINT: LPRINT
1940 LPRINT "lateral range                                    ",LR(J)
1950 LPRINT "probability of detection                         ",PP(J)
1960 LPRINT "adjusted ping cycle time (seconds)               ",CTA(J)
1970 LPRINT
1980 FOR I=0 TO M1(J)
1990 LPRINT "P("I") = "P1(I,J)/N3 TAB(30) "R("I") = "R1(I,J)
TAB(60) "Y("I") = "Y1(I,J)
2000 NEXT I
2010 NEXT J
2020 PRINT
2030 INPUT "print range sector conditional probabilities
(Y=yes/N=no)";D$
2040 IF D$="Y" OR D$="y" GOTO 2060
2050 IF D$="N" OR D$="n" GOTO 2200 ELSE GOTO 2030
2060 LPRINT: LPRINT: LPRINT: LPRINT A$+" range sector conditional
probabilities"
2070 FOR J=1 TO L
2080 LPRINT: LPRINT
2090 LPRINT "lateral range                                    ",LR(J)
2100 LPRINT "probability of detection                         ",PP(J)
2110 LPRINT "adjusted ping cycle time (seconds)               ",CTA(J)
2120 LPRINT
2130 D=N3
2140 FOR I=0 TO M1(J)
2150 IF D=0 THEN P2(I,J)=0 ELSE P2(I,J)=P1(I,J)/D
2160 LPRINT "P("I") = "P2(I,J) TAB(30) "R("I") = "R1(I,J) TAB(60)
"Y("I") = "Y1(I,J)
2170 IF D>0 THEN D=D-P1(I,J)
2180 NEXT I
2190 NEXT J
2200 END
2210 GOSUB 2270
2220 IF C$="G" OR C$="g" GOTO 2250
2230 Q=SIG*(RC1-SQR(-LOG(RV)))/SQR(RC2): REM random signal excess
part
2240 RETURN
2250 Q=SIG*SQR(-2*LOG(RV))*SIN(2*PI*RND): REM random signal
excess part
2260 RETURN
2270 RV=RND: IF RV=0 THEN RV=2.8E-38
2280 IF RV=1 THEN RV=1-2.8E-38
2290 RETURN
```

Appendix 3. A Heuristic Argument

The following heuristic argument applies to the lambda-sigma jump process. Suppose that SE is constant over a time interval of length t. Then, the occurrence of a jump during the interval that is sufficient for conditionally classifying an echo as an echo (a success) is determined by a Poisson process with rate $\Phi(SE/\sigma)/\tau$. In addition, suppose $\Phi(SE/\sigma)$ is small enough so that the probability that one success will occur in the interval is approximately $\Phi(SE/\sigma)\cdot(t/\tau)$. (If this expression is equal to .1, the probability of one success in the interval is .0905 and the probability of two or more successes in the interval is .0047.) The probability that the time to the first jump following a success will be greater than or equal to s is $\exp(-s/\tau)$. This implies that the probability of a success in the interval after which the random component of the signal excess is constant for a time at least equal to s is approximately $\Phi(SE/\sigma)\cdot(t/\tau)\cdot\exp(-s/\tau)$. This expression is a maximum when $\tau = s$. Let s be the time required for k consecutive returns and let t be a time greater than but of the order of the time required for n consecutive returns. Then, given that the signal excess at the beginning of a time interval of length t is less than zero, the above suggests that for a k-out-of-n criterion the conditional probability of detection during the interval will have a maximum for τ in the neighborhood of s if $\Phi(SE/\sigma)\cdot(t/\tau)\cdot\exp(-s/\tau)$ is small and s is sufficiently smaller than t. This suggests, for τ in the

neighborhood of s, that the range sector conditional probabilities for an encounter will have a maximum and consequently the detection probability for the encounter will have a maximum also.

As τ becomes small, $\Phi(SE/\sigma)\cdot(t/\tau)\cdot\exp(-s/\tau)$ will become greater than one and the approximation described above clearly does not hold. In this case, given the signal excess at the beginning of a time interval of length t is less than zero, the conditional probability of detection during an interval approaches the probability of 3 or more successes of out of 5 independent trials where the probability of success is $\Phi(SE/\sigma)$.

As τ becomes large, the probability of the random component of the signal excess changing during an encounter becomes small and in the limit the random component of the signal excess will be a constant value during an encounter and the signal excess can be represented by $SE + X$ where the value of X is determined at the beginning of the encounter. In this case, if $SE + X$ is greater than zero for 3 returns out of 5 during an encounter, detection will occur, otherwise it will not.

For the encounters that were considered, for both small and large values of τ, the conditional probabilities of detection for the range segments of an encounter appear to be dominated by those for τ in the neighborhood of 3 minutes.

References

1. Forrest, R. N., "An Active Acoustic Detection Model Analysis," NPS55-86-024, Naval Postgraduate School, Monterey, CA 93943-5000, December 1986.

2. McCabe, B. J. and Belkin, B., "A Comparison of Detection Models Used in ASW Operations Analysis," Daniel H. Wagner Associates, Paoli, PA 19301, October 31, 1973.

3. Miller, D. H. and Kelsey, D. W., "Determining Task Group Acoustic Coverage," CNA 76-0346, Center for Naval Analysis, Arlington, VA 22209, March 2, 1976.

4. "APSURF MOD1 ASW PROGRAMS SURFACE SHIP ENGAGEMENT MODEL (U) ABSTRACT," SAO Report 69-16, ASW Systems Project Office, Department of the Navy, Washington, D. C. 20360, January 1970.

5. Hoffman, D. W., "LORA: A Model for Predicting the Performance of Long-Range Active Sonar Systems," NUC TP 541, Naval Undersea Research and Development Center, San Diego, CA 92132, December 1976.

6. Andrews, F. A., "Two 50% Probabilities," The Journal of the Acoustical Society of America, Vol. 57, No. 1, January 1975.

7. Cavanaugh, R. C., "Acoustic Fluctuation Modeling and System Performance Estimation," Vols I and II, SAI 79-737-WA and SAI 79-738-WA, Science Applications, Inc., 84000 Westpark Drive, McLean, VA 22101, January 1978.

8. Ross, D., "Passive Sonar Performance Calculations: Lecture 211," DONANCO, Inc., La Jolla, CA.

9. Moses, E., Galati, W. and Nicholas, N., " Model Results of the Effects of Internal Waves on Acoustic Propagation," ORI, Silver Spring, MD 20910, August 3, 1978.

10. McCabe, B. J., "The Use of Detection Models in ASW Analysis," Daniel H. Wagner Associates, Paoli, PA 19301, December 5, 1974.

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